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Yukawa hierarchy from extra dimensions and infrared fixed points

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Abstract

We discuss the existence of hierarchy of Yukawa couplings in the models with extra spatial dimensions. The hierarchical structure is induced by the power behavior of the cutoff dependence of the evolution equations which yield large suppressions of couplings at the compactification scale. The values of coupling constants at this scale can be made stable almost independently of the initial input parameters by utilizing the infrared fixed point. We find that the Yukawa couplings converge to the fixed points very quickly because of the enhanced energy dependence of the suppression factor from extra dimensions as well as in the case of large gauge couplings at high-energy scale.

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It has long been one of the challenging subjects why the Nature provides very small ratios of physical parameters. A typical example is seen in the Yukawa-Higgs couplings which are small compared with the strong gauge coupling, except for the top quark, and show quite apparently a hierarchical structure between generations. There are also many examples of coupling terms which should be hierarchically suppressed, for example, neutrino masses, R-parity violating interactions, and so forth. The perturbative analysis shows that the coupling constants vary as functions of energy scale by radiative corrections. However, in ordinary four-dimensional theories, there emerges no large hierarchy because the corrections are only logarithmically dependent on the energy scale. Therefore the observed hierarchies must be given by the initial conditions at the high-energy scale.

Recently various phenomenological problems have intensively been studied in the models with extra dimensions beyond the usual four dimensions [1]–[10]. In these attempts, the coupling constants show power-law running behavior by the contributions from Kaluza-Klein modes if gauge and matter fields live in the extra dimensions [3]–[7]. Using this power-law behavior, we could obtain the hierarchical structures of couplings observed in the low-energy region. It seems, however, that the low-energy coupling constants largely depend on the initial conditions at the cutoff scale and the predictability of models is lost due to the steep changes with the energy scale.

In this letter, we discuss the existence of hierarchical Yukawa couplings stabilized by infrared fixed points within the framework of theories with extra dimensions. We also find the possibility of applying the fixed point approach even in asymptotically free gauge theories, though in four dimensions such a situation usually occurs only in asymptotically non-free theories.

Since we are interested in the possibility of large Yukawa hierarchy in the extra dimension scenarios, we work within a generic setup and investigate its infrared fixed point structure. Our setup covers the situations that the matter and gauge fields live even on the branes spreading in different number of dimensions. Once the field configuration is specified, we can write down the renormalization group equations of the system and calculate the low-energy values of running coupling constants. In this letter, we assume the generic form of renormalization group equations, not

referring to a specific model. The model-dependent details and the applications to the mass hierarchy between generations will be discussed in [11].

Before demonstrating the evolution of couplings in the extra dimensions, let us briefly review the renormalization group equations in a simple four-dimensional supersymmetric model containing superfields Ψ_1, Ψ_2 and Ψ_3 which can couple to a gauge multiplet with the coupling g, and we assume the following superpotential

$$W = y\Psi_1\Psi_2\Psi_3. \tag{1}$$

The renormalization group equations of this system with a cutoff Λ can be written as

$$\frac{d\alpha}{dt} = -\frac{b}{2\pi}\alpha^2, \tag{2}$$

$$\frac{d\alpha_y}{dt} = \frac{\alpha_y}{2\pi}(c\alpha - a\alpha_y), \tag{3}$$

where

$$\alpha \equiv \frac{g^2}{4\pi}, \quad \alpha_y \equiv \frac{y^2}{4\pi}, \quad t = \ln\frac{\Lambda}{\mu}. \tag{4}$$

Note that the coefficients b and c (≥ 0) are constants of order one expressed by the group indices, and a (> 0) denotes the multiplicity of the Yukawa coupling y in the anomalous dimensions. The system is asymptotically free (asymptotically non-free) for $b < 0$ ($b > 0$). The analytic solutions of these differential equations are obtained,

$$\alpha(t) = \frac{\alpha(0)}{1 + \frac{b}{2\pi}\alpha(0)t}, \tag{5}$$

$$\alpha_y(t) = \frac{\alpha_y(0)E(t)}{1 + \frac{a}{2\pi}\alpha_y(0)F(t)}, \tag{6}$$

where the functions $E(t)$ and $F(t)$ are defined as

$$E(t) = \left(1 + \frac{b}{2\pi}\alpha(0)t\right)^{c/b} = \left(\frac{\alpha(0)}{\alpha(t)}\right)^{c/b}, \tag{7}$$

$$F(t) = \int_0^t E(t')dt'. \tag{8}$$

In order to analyze a fixed point structure, now we define the ratio of couplings R. From Eqs. (2) and (3), we have

$$\frac{dR}{dt} = \frac{-1}{2\pi}(b + c)\alpha R(R - 1), \tag{9}$$

$$R \equiv \frac{a}{b+c}\frac{\alpha_y}{\alpha}. \tag{10}$$

That implies that $R^* = 1$ is the infrared fixed point. Solving Eq. (9), we obtain

$$\frac{R(t)-1}{R(t)} = \xi \frac{R(0)-1}{R(0)}, \tag{11}$$

where ξ is defined as

$$\xi = \frac{1}{E(t)}\left(\frac{\alpha(t)}{\alpha(0)}\right). \tag{12}$$

The suppression factor ξ is written only by the gauge coupling constant and provides the rate at which R approaches to the infrared fixed point value $R^* = 1$, which has been discussed to demonstrate the fast convergency in the asymptotically non-free gauge models [12]. Using Eq. (7), we have $\xi = (\alpha(t)/\alpha(0))^{1+c/b}$ which shows that when $b > 0$, $\xi \to 0$ very rapidly with increasing t (in the infrared). On the other hand, ξ is not so suppressed in the case of asymptotically free theories ($b < 0$) and then the predictions from the fixed point approach cannot be reliable.

Now let us consider a gauge-Yukawa system in higher dimensional spacetime assuming that the chiral superfields as well as the gauge multiplet can live in $4+\delta$ dimensions with δ dependent on each field. The number of dimensions of the extended space may be more than one and it becomes even 6 in string theories. Thus each compactification scale may be different from the others, but we here suppose that for simplicity, all the compactification scales are equal to μ_0 although the following analyses can be applied to generic cases straightforwardly. Above the compactification scale μ_0, the contributions of the Kaluza-Klein modes come into play. Neglecting the logarithmic terms from the contributions of ordinary four-dimensional particles, the truncated 1-loop effective renormalization group equations are written as [4]

$$\frac{d\alpha}{dt} = -\frac{\widetilde{b}}{2\pi}X_{\delta_g}\left(\frac{\Lambda}{\mu}\right)^{\delta_g}\alpha^2 + \cdots, \tag{13}$$

$$\frac{d\alpha_y}{dt} = \frac{\alpha_y}{2\pi}\sum_{i=1,2,3}\gamma_i, \tag{14}$$

where γ_i denote the anomalous dimensions of Ψ_i,

$$\gamma_i = \widetilde{c}_i X_{\delta_{g_i}}\left(\frac{\Lambda}{\mu}\right)^{\delta_{g_i}}\alpha - \widetilde{a}_i X_{\delta_i}\left(\frac{\Lambda}{\mu}\right)^{\delta_i}\alpha_y + \cdots \tag{15}$$

with the cutoff Λ, above which effects of new physics become important. In the above equations, the power factors appeared in the beta functions originate from the Kaluza-Klein modes propagating in the loops. The δ_g part corresponds to the largest gauge contribution to the anomalous dimension of gauge fields and the ellipses denote less dominant terms with smaller powers, which have negligible effects on the evolution of couplings. Similarly, δ_{g_i} and δ_i are the largest gauge and Yukawa contributions, respectively, to the anomalous dimension of matter fields. These exponents can be determined once we fix the configuration of relevant fields in the extra dimensions. The volume factor $X_\delta = \pi^{\delta/2}/\Gamma(1+\delta/2)$ is originated from the phase-space integral of Kaluza-Klein modes.* In the following, we redefine the coefficients in the beta functions for simplicity as $a_i = \tilde{a}_i X_{\delta_i}$, $b = \tilde{b} X_{\delta_g}$, and $c_i = \tilde{c}_i X_{\delta_{g_i}}$.

Our goal is to show how the hierarchy of Yukawa couplings is realized as a result of their running effects from Λ to μ_0. Thus μ_0 is regarded as the energy scale at which we expect that Yukawa couplings are determined almost independently of the initial values at Λ. This may be possibly realized if they have infrared fixed points. For that purpose, we now investigate the fixed point structure at the Kaluza-Klein threshold scale μ_0. From Eqs. (13) and (14), we have

$$\frac{d}{dt} \ln \frac{\alpha_y}{\alpha} = -\frac{1}{2\pi} \left[a \left(\frac{\Lambda}{\mu}\right)^{\delta_y} \alpha_y - \left\{ b \left(\frac{\Lambda}{\mu}\right)^{\delta_g} + c \left(\frac{\Lambda}{\mu}\right)^{\delta_{g'}} \right\} \alpha \right] \tag{16}$$

with $\delta_y = \mathrm{Max}(\delta_i)$ and $\delta_{g'} = \mathrm{Max}(\delta_{g_i})$, and a and c are the corresponding coefficients. By the analogy with the four-dimensional case, we define

$$R \equiv \frac{a(\Lambda/\mu)^{\delta_y}}{b(\Lambda/\mu)^{\delta_g} + c(\Lambda/\mu)^{\delta_{g'}}} \cdot \frac{\alpha_y}{\alpha}, \tag{17}$$

then the renormalization group equation for R becomes

$$\frac{dR}{dt} \simeq -\frac{1}{2\pi} \left[b \left(\frac{\Lambda}{\mu}\right)^{\delta_g} + c \left(\frac{\Lambda}{\mu}\right)^{\delta_{g'}} \right] \alpha R(R-1). \tag{18}$$

This equation shows that $R^* = 1$ is the infrared stable fixed point of this model as long as R is positive. It should be noted that while R^* is the constant, the

*In Ref. [7], it has been obtained $X_\delta = \pi^{\delta/2}/\Gamma(2+\delta/2)$. The difference can be absorbed by the redefinition of the cutoff Λ for $\delta_i = \delta_g$. Our results are irrelevant to the explicit value of X_δ.

'fixed point' value of α_y/α is generally energy dependent. In the above equation, we have neglected the sub-leading terms suppressed by the small value of μ/Λ. If one includes these terms, the fixed point value $R^* = 1$ receives a small correction suppressed by μ/Λ but then the convergence behavior to that fixed point by the factor ξ is unchanged. In other words, even when one redefines R such that $R \to 1$ in the infrared, this change has little effect on the suppression factor ξ.

We can find how fast R approaches to the infrared fixed point by solving the above renormalization group equation. The solutions of α_y, R and the suppression factor ξ obtained in the four-dimensional case ((6), (11) and (12)) still hold in the extra dimension scenarios if we now replace the definitions of $E(t)$ and $F(t)$ with

$$E(t) = \exp\left[\int_0^t dt' \, \frac{c}{2\pi}\left(\frac{\Lambda}{\mu}\right)^{\delta_{g'}} \alpha\right], \tag{19}$$

$$F(t) = \int_0^t dt' \left(\frac{\Lambda}{\mu}\right)^{\delta_y} E(t'). \tag{20}$$

It is easily seen that in the case of $\delta_y = \delta_{g'} = \delta_g$, the forms of the solutions are identical to those in four dimensions with a suitable change of the variable t.

Since the couplings α and α_y have power running behaviors, the running region between Λ and μ_0 may be much narrower than the four-dimensional case. Therefore if we make use of the infrared fixed points, the strong convergence to them is required. We now examine the behavior of the suppression factor $\xi = E(t)^{-1}(\alpha(t)/\alpha(0))$ when the power factors δ_y, δ_g and $\delta_{g'}$ are varied. First, it is noted that ξ is determined only from the informations of the gauge sector, $\delta_{g,g'}$ and α, but does not depend on δ_y [6]. This means that the convergency to the infrared fixed point is dominated by the gauge coupling constant (except for the large $\alpha_y(0)$ case). In Fig. 1, we show the dependence of the suppression factor ξ on $\delta_{g'}/\delta_g$ and b. From this figure, one can see that there are two distinct ways to have a small value of ξ. One is realized when $b > 0$, namely in the asymptotically non-free theories. This is clearly seen when the effect from the gauge anomalous dimension is dominant in the evolution of R ($\delta_{g'}/\delta_g < 1$). The situation is almost similar to the ordinary four-dimensional cases [12]. The other possibility is essentially due to the existence of the extra dimensions. When the gauge contributions of the matter anomalous dimensions govern the renormalization group equation ($\delta_{g'}/\delta_g > 1$), the suppression

factor ξ becomes very small even in the asymptotically free theories such as the minimal supersymmetric standard model. It means that R approaches its fixed-point value very quickly in the infrared. This is in sharp contrast to the four-dimensional case. Figure 2 shows that R indeed converges to the infrared fixed point ($R^* = 1$) within the region $\Lambda/\mu < O(10)$ for a wide range of initial Yukawa couplings. If ξ approaches to 0 rapidly enough, on the infrared point μ_0 we have a relation between $\alpha_y(\mu_0)$ and $\alpha(\mu_0)$ independently of the high-energy input values.

$$\alpha_y^*(\mu_0) \;\; = \;\; \left[\frac{b}{a}\left(\frac{\mu_0}{\Lambda}\right)^{\delta_y-\delta_g} + \frac{c}{a}\left(\frac{\mu_0}{\Lambda}\right)^{\delta_y-\delta_{g'}}\right]\alpha^*(\mu_0). \tag{21}$$

Thus we can obtain a variety of hierarchical power factors for μ_0/Λ in the infrared fixed points of Yukawa couplings according to the values of δ's.

When ξ is not so small, by contrast, the system flows slowly to the fixed point and then Eq. (11) shows that the initial $R(0)$ dependence of $R(t)$ generally spoils the predictions from the infrared fixed points. However, even in this case, if $R(0)$ is large enough, the $R(0)$ dependence on the right-handed side of Eq. (11) disappears and the predictability is recovered. In this case, we have

$$R^*(\mu_0) \;\; = \;\; \frac{1}{1-\xi(\mu_0)}. \tag{22}$$

This corresponds to the so-called quasi fixed point [13]. As well as in the true fixed point approach, we can still obtain a hierarchical structure of Yukawa couplings from the fixed points, i.e., independently of the high-energy input parameters.

So far, we have considered the simple model with only one Yukawa coupling. We will extend the model to contain more numbers of Yukawa couplings and show that a large hierarchy is indeed generated. For simplicity, consider a case with two independent Yukawa couplings α_{y_1} and α_{y_2}. As seen from Eq. (21), the hierarchy among them comes from different choices of δ's. In Fig. 3, we show an example of possible hierarchy in this two generation model with $\delta_g = 1$, $\delta_{g'_1,g'_2} = 2$, $\delta_{y_1} = 1$, and $\delta_{y_2} = 4$. Since we take $\delta_{g'_i} > \delta_g$, the infrared fixed points are actually realized even in $b < 0$ (asymptotically free case). The model gives rise to a large hierarchy $\alpha_{y_1}/\alpha_{y_2} \sim 10^4$ at the Kaluza-Klein threshold μ_0 due to the difference of δ_y. As discussed above, these values correspond to the fixed points and not dependent on the initial values $\alpha_{y_i}(0)$. It is straightforward to extend the above discussions to the

models with more numbers of Yukawa couplings. This interesting possibility may be readily applied to the quarks and leptons mass hierarchy between generations and to other phenomenological problems involving very different order of coupling constants [11].

Here a few comments on the available value of Λ/μ_0 are in order. In Ref. [6], the relation between the order of hierarchy factor and the perturbation limit was emphasized. They concluded that if one uses the infrared fixed points to have hierarchical Yukawa couplings, one is forced to adopt the strong unification scenarios and then the perturbative reliability restricts the hierarchy factor to less than the order of 10. However, we have seen in the above that the strong convergency to the Yukawa fixed points can be accomplished not only in the case of the strong gauge couplings but also $\delta_{g'}/\delta_g > 1$. The latter is realized of course even in asymptotically free models and thus, the perturbation limit can be rather relaxed. Secondly, if one tries to incorporate the fixed point scenario with the gauge coupling unification, it seems natural to identify Λ to the unification scale. In that case, the largest value of available hierarchy is roughly estimated as

$$\left(\frac{\Lambda}{\mu_0}\right)^{\delta_g} \sim \ln\left(\frac{M_{\rm GUT}}{M_W}\right) \sim 30, \tag{23}$$

which also constrains the value of Λ/μ_0, and we may require a small δ_g in order to obtain an enough large Λ/μ_0 for the quarks and leptons mass differences. After all, to satisfy the above two limits one needs a smaller value of δ_g compared to other power factors. This could be achieved by some cancellations between diagrams, the specific field configurations in the extra dimensions, and so on. It should be noted that the asymmetrical compactifications of extra dimensions can also lead a various orders of hierarchy factors from infrared fixed points even with the common values of δ's.

We have studied the infrared fixed points behaviors of the renormalization group equations of a simple gauge-Yukawa model with extra dimensions. It is found that the hierarchies among the Yukawa couplings can be realized as the fixed point values. The fixed-point structures largely depend on the field configurations in the extra dimensions (δ_i, δ_g and δ_{g_i}). Under certain conditions concerning the extra dimensions, one can observe the strong attraction to the infrared fixed points and then

it can be applicable to various models in which hierarchical structure of couplings is needed. Though there might exist some subtleties in applying it to phenomenological models, we hope this approach will give a possible solution to unsolved problems in particle physics.

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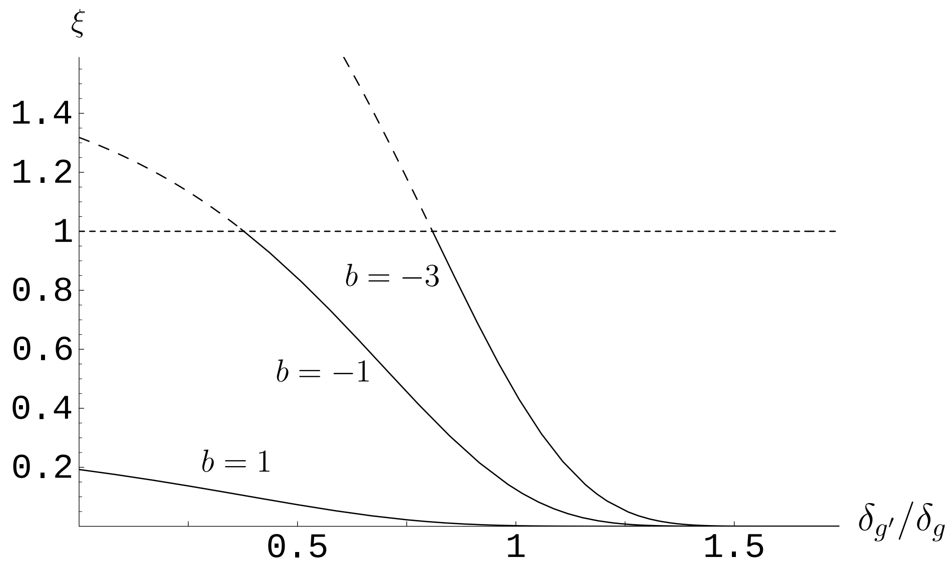

Figure 1: The typical behaviors of the suppression factor ξ. Above the horizontal dotted line $\xi = 1$, the fixed-point solution becomes infrared unstable. $\delta_{g'}/\delta_g = 1$ recovers the four-dimensional values of ξ. We set $\alpha(t) = 0.1$ and $c = 16/3$.

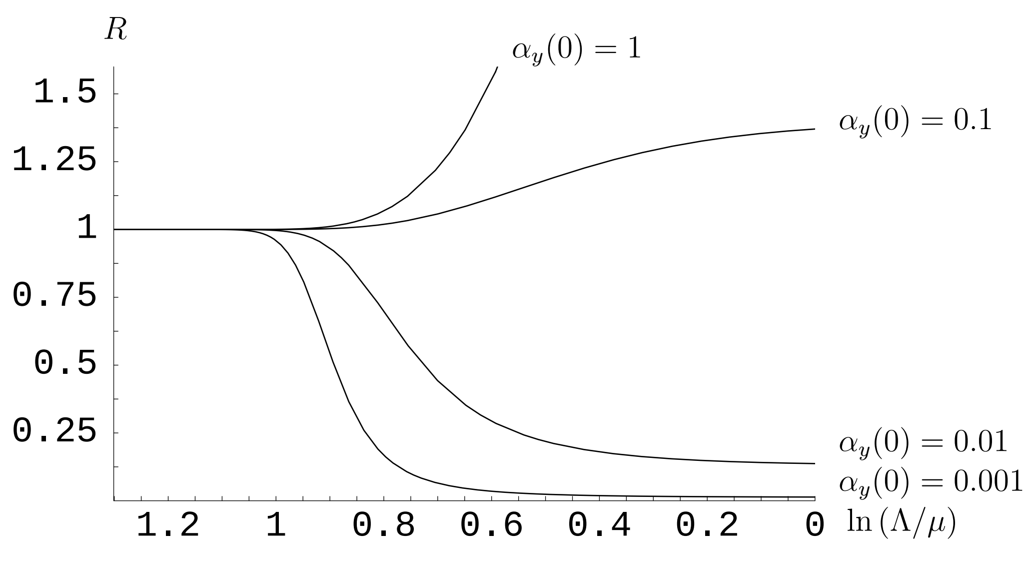

Figure 2: The renormalization group evolutions of R approaching to the fixed point $R^* = 1$ with various initial conditions of α_y. We set $\delta_g = 1$, $\delta_{g'} = \delta_y = 2$, $a = 3$, $b = -3$, and $c = 16/3$.

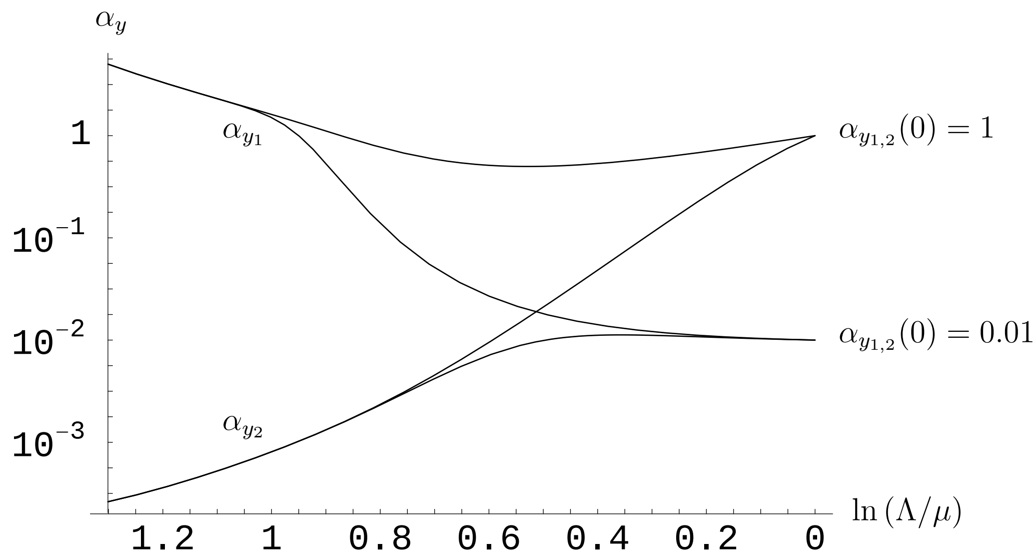

Figure 3: The fixed point hierarchy of Yukawa couplings in the two generation model. The initial values of two couplings at $\mu = \Lambda$ are taken to be equal, $\alpha_{y_{1,2}}(0) = 1$ or 0.01. We set $\delta_{y_1} = 1$ and $\delta_{y_2} = 4$ and the other beta functions are same as in Fig. 2.